

# Flavor Singlet Axial Coupling of the Proton – An Updated Analysis \*

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We present a combined analysis of SESAM and  $T\chi L$  data for the flavor singlet axial coupling  $G_A^1$  of the proton, which is very helpful to stabilize the disconnected signals at small quark masses. From connected and disconnected contributions we use the tadpole improved renormalization constant  $Z_A$  and obtain  $G_A^1 = 0.21(12)$ .

## 1. Introduction

The flavor singlet axial coupling of the proton is defined as

$$s_\mu G_A^1 = \langle P | \bar{u} \gamma_\mu \gamma_5 u + \bar{d} \gamma_\mu \gamma_5 d + \bar{s} \gamma_\mu \gamma_5 s | P \rangle, \quad (1)$$

where  $s_\mu$  denotes the components of the proton polarization vector. In the naive parton model  $G_A^1$  is related to the fraction of the proton spin carried by the quarks. From the measurement of the first moment of the spin dependent proton structure function,  $g_p^1$ , in deep inelastic polarized muon proton scattering the EMC experiment [1] found a small value,

$$G_A^1 = 0.12(17) \quad (2)$$

which led to the “proton spin crisis”. The result for  $G_A^1$  indicates that the contribution to the proton spin from the quarks is small. New experiments, including proton, neutron and deuteron data [2], have shifted the value up to  $G_A^1 = 0.29(6)$  which is still far away from the Ellis-Jaffe QCD sum-rule prediction [3]:

$$G_A^1 \simeq G_A^8 = 0.579(25) \quad (3)$$

Albeit a lot of theoretical and experimental work has been done in the meantime, a clear understanding of the proton spin is still lacking. In this context lattice calculations are very illuminating though hampered by large fluctuations [4].

## 2. Lattice techniques

On the lattice both connected and disconnected diagrams for  $G_A^1$  (see figure 1) can be calculated from ratios of correlation functions [4].

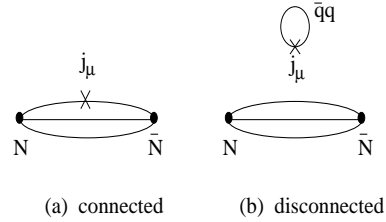


Figure 1. *Connected (a) and disconnected (b) diagrams contribute to the flavor singlet axial coupling  $G_A^1$  of the proton ( $j_\mu = \bar{q} \gamma_\mu \gamma_5 q$ ).*

The SESAM collaboration has analyzed  $200 \cdot 16^3 \times 32$  gauge configurations at each of their four quark masses corresponding to  $m_\pi/m_\rho = 0.833 \dots 0.686$ . The configurations have been generated previously in the standard Wilson discretization scheme with  $n_f = 2$  mass degenerate quark flavors, at  $\beta = 5.6$ . In addition 200  $T\chi L$  gauge configurations ( $24^3 \times 40$  lattices) at the same coupling  $\beta = 5.6$  have been analyzed. The quark mass of the  $T\chi L$  lattices corresponds to the lightest SESAM quark mass. Details of the simulations can be found in [5,6].

### 2.1. Connected contributions

For the amplitudes from the connected diagrams (see figure 1(a)) in the matrix element, eq. 1, we have applied the global summation method and the insertion technique as described in [4].

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From the ratio

$$R_{A_\mu}^{SUM} = \frac{\sum_{\vec{x}} \langle P^\dagger \sum_{\vec{y}, y_0} [\bar{q} \gamma_\mu \gamma_5 q] (\vec{y}, y_0) P \rangle}{\sum_{\vec{x}} \langle P^\dagger P \rangle} - \langle \sum_{\vec{y}, y_0} [\bar{q} \gamma_\mu \gamma_5 q] (\vec{y}, y_0) \rangle, \quad (4)$$

where  $P$  is an interpolating operator for the proton, the connected amplitude  $C_q = \langle P | \bar{q} \gamma_\mu \gamma_5 q | P \rangle_{conn}$  can be obtained from the asymptotic linear slope

$$R_{A_\mu}^{SUM}(t) \xrightarrow{t \rightarrow \infty} A + \langle P | \bar{q} \gamma_\mu \gamma_5 q | P \rangle t. \quad (5)$$

## 2.2. Disconnected contributions

For the disconnected amplitudes  $D_q = \langle P | \bar{q} \gamma_\mu \gamma_5 q | P \rangle_{disc}$  (see figure 1(b)) we have used the plateau accumulation method (PAM) [4]: from the partial summation

$$R_{A_\mu}^{PAM}(t, \Delta t_0, \Delta t) = \sum_{y_0=\Delta t_0}^{t-\Delta t} R_{A_\mu}^{PLA}(t, y_0), \quad (6)$$

with

$$R_{A_\mu}^{PLA} = \frac{\sum_{\vec{x}} \langle P^\dagger \sum_{\vec{y}} [\bar{q} \gamma_\mu \gamma_5 q] (\vec{y}, y_0) P \rangle}{\sum_{\vec{x}} \langle P^\dagger P \rangle} - \langle \sum_{\vec{y}} [\bar{q} \gamma_\mu \gamma_5 q] (\vec{y}, y_0) \rangle. \quad (7)$$

$D_q$  follows from the asymptotic time dependence ( $t \rightarrow \infty$ ) in

$$R_{A_\mu}^{PAM}(t, \Delta t_0, \Delta t) = B + D_q(t - \Delta t - \Delta t_0). \quad (8)$$

For the calculation of the axial vector quark loops we have used the spin explicit stochastic estimator technique with complex Z2 noise and 100 stochastic estimates per spin component and gauge field configuration [4].

## 3. Raw data

Figure 2 displays  $R_A^{SUM}(t)$  for the connected amplitudes  $C_{u,d}$  on the four SESAM quark masses. The results from the linear fits are summarized in table 1. For the lightest SESAM quark mass ( $\kappa_{sea} = 0.1575$ ) we have also calculated  $C_{u,d}$  on the  $T\chi L$  lattice. Note that the volume effects are less than 7%.

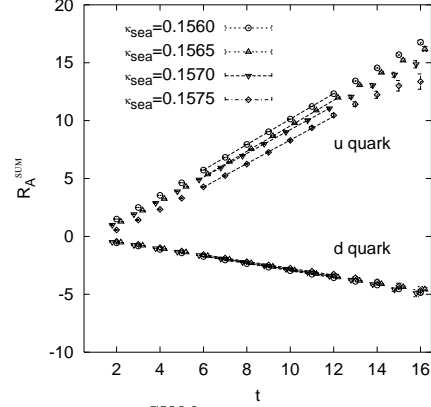


Figure 2.  $R_A^{SUM}(t)$  for the SESAM gauge-configurations (spin-averaged). The linear fits give the connected amplitudes  $C_u$  and  $C_d$ .

$\kappa$	$C_u$	$C_d$
0.1560	1.100(8)	-0.307(3)
0.1565	1.102(11)	-0.308(4)
0.1570	1.025(12)	-0.297(8)
0.1575(SESAM)	1.018(24)	-0.295(10)
0.1575( $T\chi L$ )	1.086(11)	-0.310(6)

Table 1

Connected amplitude  $C_{u,d}$  from the linear fits to  $R_A^{SUM}(t)$  (SESAM and  $T\chi L$ ).

In Figure 3 we plotted  $R_A^{PAM}(t)$  ( $\Delta t = \Delta t_0 = 1$ ) for the disconnected amplitudes with  $\kappa_{sea} = 0.156$  (SESAM) and  $\kappa_{sea} = 0.1575$  ( $T\chi L$ ). Albeit the statistical errors for  $D_q$  are large we observe comparable quality of signals from both plots.

## 4. Results

The chiral extrapolations of  $C_{u,d}$  and  $D_{q,s}$  to the light quark mass  $m_{light}$  are shown in figure 4. Since the finite volume effects are small we have taken the  $T\chi L$  data at  $\kappa_{sea} = 0.1575$  for the extrapolation in  $D_q$ . Within the errors the flavor symmetry  $D_q = D_s$  in the disconnected amplitudes is preserved and we use only the connected amplitudes for  $G_A^8$  and  $G_A^3$ .

With the Wilson discretization both the flavor singlet and non-singlet current need renormalization. We have used the first order (tadpole im-

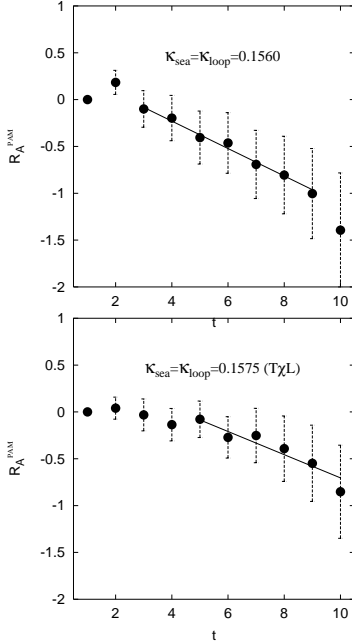


Figure 3.  $R_A^{PAM}(t)$  for  $\kappa_{sea} = 0.156$  (SESAM) and  $\kappa_{sea} = 0.1575$  (T $\chi$ L) (spin-averaged). The linear fits give the disconnected amplitude  $D_q$ .

proved) result from lattice perturbation theory for  $Z_A^S$  and  $Z_A^{NS}$ [4]. The renormalized coupling constants of the proton and the contributions  $\Delta q$  to the proton spin are listed in table 2.

$\Delta u$	$\Delta d$	$\Delta s$
0.62(7)	-0.28(6)	-0.12(7)
$G_A^1$	$G_A^3$	$G_A^8$
0.21(12)	0.907(20)	0.484(18)

Table 2

*Renormalized coupling constants of the proton and the contributions  $\Delta q$  to the proton spin.*

## 5. Conclusion

We have calculated connected and disconnected contributions to the flavor singlet axial vector coupling of the proton in a full QCD  $n_f = 2$  lattice simulation with Wilson fermions. We have checked the volume effects for the connected amplitudes when we increase the lattice

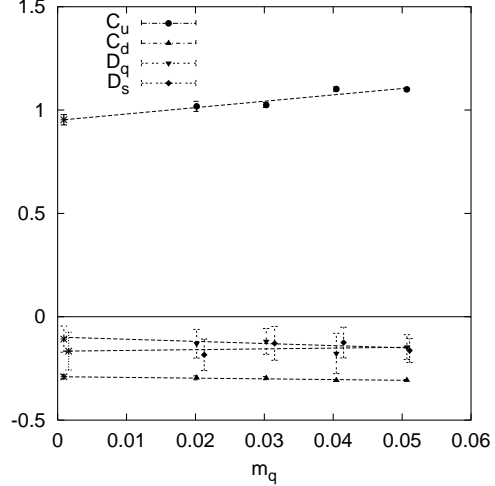


Figure 4. Linear chiral extrapolations of  $C_{u,d}$  and  $D_{q,s}$  to the light quark mass  $m_{light}$ .

size at the same coupling ( $\beta = 5.6$ ) and quark mass. In the analysis of the disconnected contributions we have replaced the SESAM results for the lightest quark mass by the T $\chi$ L data and obtained a better signal for  $D_q$ . Nevertheless, the errors are dominated by the large statistical fluctuations in the disconnected amplitudes. We find  $G_A^1 = 0.21(12)$  to be consistent with the result from experiment and with previous quenched estimates [7,8]. For the triplet coupling we get  $G_A^3 = 0.907(20)$  which is 30% smaller than the experimental value,  $G_A^3 = 1.2670(35)$  [2]. To fix the systematic errors in the lattice calculation we would need a scaling analysis as well as a non-perturbative determination of  $Z_A^{NS}$  and  $Z_A^S$ .

## REFERENCES

1. EMC, J. Ashman et al., Nucl. Phys. **B328** (1989) 1; Phys. Lett. **B206** (1988)364.
2. SMC, D. Adams et al., Phys. Rev. **D56** (1997)5330.
3. J. Ellis, R.L. Jaffe, Phys. Rev. **D9** (1974) 1444; Phys. Rev. **D10** (1974)1669.
4. SESAM collaboration, hep-lat/9901009, appears in Phys.Rev. **D**.
5. SESAM/T $\chi$ L collaboration, Nucl.Phys. Proc.Supp. **60A** (1998) 311.

6. SESAM Collaboration, Phys.Rev. **D59**, 014509 (1999).
7. M. Fukugita, Y. Kuramashi, M. Okawa, A. Ukawa, Phys. Rev. Lett. **75** (1995) 2092.
8. S.J. Dong, J.F. Lagaë, K.F. Liu, Phys. Rev. Lett. **75** (1995) 2096.